

S. Mai Real Analysis 2019/2020 නියම තුළ ආකෘතිවාස්ථාන Realno. සංඛ්‍යා සේවක දෙපාර්තමේන්තු මධ්‍ය මධ්‍යම සැප්තෝමැබර් 2020 පෙරින්දී සෞඛ්‍යා අභ්‍යන්තර ප්‍රාග්ධනයට නොවා.

වෙත ඇති ප්‍රාග්ධන නිශ්චිත විශ්චාලී නිශ්චාලී සංඛ්‍යා නිශ්චාලී Natural number 1, 2, 3, ... a finite sequence

සූත්‍රමය සංඛ්‍යා 1, 2, 3, ..., n නිශ්චාලී නිශ්චාලී නිශ්චාලී set

නිශ්චාලී සිංහල නිශ්චාලී නිශ්චාලී නිශ්චාලී N.

සෑම සිංහල නිශ්චාලී නිශ්චාලී නිශ්චාලී නිශ්චාලී

අභ්‍යන්තර නිශ්චාලී නිශ්චාලී නිශ්චාලී

- - - - 3, - 2, - 1 නිශ්චාලී නිශ්චාලී නිශ්චාලී (total)

ඕනෑම නිශ්චාලී, නිශ්චාලී නිශ්චාලී නිශ්චාලී නිශ්චාලී

සෑම සිංහල නිශ්චාලී නිශ්චාලී නිශ්චාලී

(i.e. the set of all integers) නිශ්චාලී නිශ්චාලී

අභ්‍යන්තර නිශ්චාලී නිශ්චාලී නිශ්චාලී

$Q = \left\{ \frac{p}{q} : p, q \in \mathbb{Z}, q \neq 0 \text{ and } (p, q) = 1 \right\}$.

එහි නිශ්චාලී නිශ්චාලී නිශ්චාලී

අභ්‍යන්තර නිශ්චාලී නිශ්චාලී නිශ්චාලී

irrational numbers. නිශ්චාලී නිශ්චාලී නිශ්චාලී

එහි නිශ්චාලී නිශ්චාලී නිශ්චාලී

ගැනීමේ නිශ්චාලී නිශ්චාලී

The collection of all rational and irrational numbers is

called the set of real numbers denoted by R.

So, you can verify that $N \subset Q \subset R$. Also $Q^c \subset R$

In fact $Q \cup Q^c = R$. [~~ගැනීමේ නිශ්චාලී~~ ~~ගැනීමේ නිශ්චාලී~~]

අභ්‍යන්තර නිශ්චාලී නිශ්චාලී නිශ්චාලී

එහි නිශ්චාලී නිශ්චාලී නිශ්චාලී

i) නිශ්චාලී නිශ්චාලී නිශ්චාලී

ii) නිශ්චාලී නිශ්චාලී නිශ්චාලී

are

for pure Analysis නිශ්චාලී නිශ්චාලී

Contd.

Some properties of real numbers.

A. Let $(\mathbb{R}, +, \cdot)$ be an algebraic structure where \mathbb{R} is the set of all real numbers and $+$, \cdot are two binary compositions defined on \mathbb{R} . Then we have the following properties. $+$ is addition, \cdot is multiplication

$$\text{1.a) } x+y = y+x \quad \& \quad \text{1.b) } x \cdot y = y \cdot x \quad \forall x, y \in \mathbb{R}.$$

This is commutative law.

$$\text{2. Associativity law } \forall x, y, z \in \mathbb{R}$$

$$\text{a) } x+(y+z) = (x+y)+z \quad \& \quad \text{b) } (x \cdot y) \cdot z = x \cdot (y \cdot z)$$

3. Existence of identity elements.

$$\text{a) } 0 \in \mathbb{R} \text{ s.t. } \forall x \in \mathbb{R}, 0+x = x+0 = x$$

3.b) $1 \in \mathbb{R}$ s.t. $\forall x \in \mathbb{R}, 1 \cdot x = x \cdot 1 = x$

0 is the additive identity and
 1 is multiplicative identity element in \mathbb{R} .

4) Existence of additive inverse in \mathbb{R} .

$$\forall x \in \mathbb{R} \exists -x \in \mathbb{R} \text{ s.t. } x+(-x) = (-x)+x = 0$$

\rightarrow is called the additive inverse of x

5) Existence of multiplicative inverse of a non-zero element in \mathbb{R} .

$$\forall x \in \mathbb{R} \text{ and } x \neq 0, \exists \frac{1}{x} \in \mathbb{R} \text{ s.t. } x \cdot \frac{1}{x} = 1$$

$\frac{1}{x}$ is the multiplicative inverse of $x \neq 0$ in \mathbb{R} .

6) Distributive law.

$\forall x, y, z \in \mathbb{R}$, the law

$$x \cdot (y+z) = x \cdot y + x \cdot z \text{ is called the distributive law.}$$

The above properties constitute the Field axioms of \mathbb{R} .

B. The order structure of \mathbb{R} .

A linear order relation ' $<$ ' read as 'less than'

and defined $x < y$ if $x \in \mathbb{R}, y \in \mathbb{R}$ and

x is less than y . This relation satisfies the conditions

i) For every pair of real numbers x, y any one

of the statements is true at a time

$x < y \Leftrightarrow x = y$ or $y < x$. This is called law of trichotomy.

i) $a < b, b < c \Rightarrow a < c$ & $a, b, c \in \mathbb{R}$.
This is called transitivity law.

$\forall x \in \mathbb{R}, x > 0$ or $x < 0$ or $x = 0$.
equivalently

x is called positive when $x > 0$,
negative when $x < 0$.

$x > y$ and $y < x$ are equivalent statements.

ii) If $x < y$ and z be any real no. Then,

$$x+z < y+z$$

If $x < y$ and $z < 0$ then
 $x+z < y+z$ ($z > 0$)

and if z is negative ($z < 0$)

$$xz > yz$$

This is called the Compatibility Condition.

The above laws i), ii) and iii) together are order properties or an order structure of real nos.

This order together with field structure we can say that $(\mathbb{R}, +, \cdot)$ is an ordered field.

Study also the absolute value and triangle inequality from any book.

Next Completeness axiom of real numbers.

It formally written very difficult, most difficult concept.
We will go to 2D.

Let S be a subset of \mathbb{R} , and $m \in \mathbb{R}$. Then m will be called an upper bound if $\forall x \in S, x \leq m$.

Again if $\exists m \in \mathbb{R}$ s.t. $\forall x \in S, x \geq m$, then m will be called a lower bound. If S has an upper bound, S is called bounded above and if S has a lower bound. If S has both upper and lower bounds, S is called a bounded set.

Q2. What is the difference between upper bound and lower bound?

- a) If \exists inf. upper bound or lower bound \forall lower or upper bound
then, there is unique upper bound (unique). And if upper bound exists,
 \exists \exists $M \in S$. If there are two or more upper bound \exists $M_1, M_2 \in S$,
then, exists inf. upper bound \exists M such that $M \leq M_1, M_2$ (using
(infinity) upper bound or every $M \in S$,
exists lower bound \exists $m \in S$ such that $m \leq M$,
(exists $m \in S$)

- b) can be bounded above, below or ~~extreme~~ bounded
bounds \exists $M, m \in S$ such that $m \leq x \leq M$. See
ex. 1

Ex. 1

under what way. S is considered a subset of R

Then what about are your opinion
about the Possibility of $S = \emptyset$, S is finite and
 S is infinite. Think independently and try to
give find answers.

- $S = \{1 \leq x \leq 2\}$ bounded set, 1 is a lower
bound and 2 is an upper bound
both belongs to S .
- Whether 0 can be its lower bound?
- $S = \{1 < x < 2\}$ "

$S = \{\frac{1}{n} : n \in \mathbb{N}\}$ upper bound 1 belongs to S
lower bound 0 doesn't $\in S$.

Next Supremum and Infimum,
and $S \subseteq R$ be bounded above.

Let $S \subseteq R$ and $M \in R$. Then M will called the Supremum
of S if i) $x \leq M \quad \forall x \in S$
ii) for a given Positive no ϵ , however small
 \exists at least one $x \in S$ satisfying
 $x > M - \epsilon$.

i.e. any real no less than M cannot be an upper bound then.

i.e. The Supremum of S is the least of all its upper bounds.
That's why it is also known as the least upper bound (lub).

Also ~~least~~ Supremum of a set is called by the upper bound.
Note. If S is not bounded above, then its supremum does not
exist.

write yourself and Post the definition of the Infimum or
the greatest lower bound (glb) / the lower bound in the same way.

Post your question on the discussion!